R&D NOTES

A Simplified Procedure to Identify Trailing Vortices Generated by a Rushton Turbine

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Introduction

The flow structure generated in the impeller region of a tank stirred by a Rushton turbine is quite complex. The impeller-induced mean flow is composed of a jet flow and two tip vortices (unbaffled vessels[1]; baffled vessels[2–4]). In addition, the flow is fully turbulent at a high Reynolds number. These vortices, named "tip vortices" or "trailing vortices," provide a source of turbulence (from Escudié, Bouyer, and Liné, hereafter referred to as EBL[5]) and have a potential benefit for mixing practice, such as optimization of tracer/reactant injection. Determining their location, size, and strength is thus relevant. The present article focuses on the identification of such vortical structures.

Because the flow is three-dimensional (3-D) and turbulent, it is necessary to develop a complex experimental procedure to achieve this objective. Whatever the experimental techniques [Laser-Doppler velocimetry, particle image velocimetry (PIV)], data acquisitions are first synchronized with the blade position to measure phase-averaged velocity components.[3,4,6–14] Before the work of EBL,[5] two approaches were commonly used to localize the trailing vortices : (1) from the phase-averaged velocity fields, for a given angular position of the measurement plane compared to the blade, the vortex center is defined by the location where the vertical velocity is nil, assuming that there is no vertical displacement of the vortices; (2) from a dimensionless vorticity (ξ), calculated in a vertical plane of measurement relative to the blade position, an identifier's threshold ξ_c is arbitrarily fixed to identify the vortex region.[11,12]

EBL used the objective definition of a vortical structure proposed by Jeong and Hussain,[15] who based their analysis on the following equation:

$$\Omega_{ik}\Omega_{kj} + S_{ik}S_{kj} = -\frac{1}{\rho}\frac{\partial^2 P}{\partial x_i \partial x_j} = -\frac{1}{\rho}p_{i,j} \tag{1}$$

where S is the deformation rate tensor (symmetrical part of the velocity gradient tensor $\nabla \otimes u$), given as

$$S_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) \tag{2}$$

and Ω is the rotation rate tensor (antisymmetrical part of $\nabla \otimes u$):

$$\Omega_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} - \frac{\partial u_j}{\partial x_i}\right) \tag{3}$$

Two positive eigenvalues of the tensor $p_{i,j}$ indicate the presence of a local pressure minimum corresponding to a vortex region. If λ_1, λ_2, and λ_3 are taken as the eigenvalues of $S^2 + \Omega^2$ and if $\lambda_1 \geq \lambda_2 \geq \lambda_3$, a vortex core corresponds to $\lambda_2 < 0$. The advantage of this method for determining the trailing vortex compared to previous techniques was highlighted by EBL.[5]

Because the nine different components of the gradient velocity tensor $\nabla \otimes u$ are needed with this approach, it is necessary to measure the three velocity components in the three directions of the frame of reference and to synchronize the data acquisitions with the blade position to investigate the flow between two successive blades. If a conventional PIV device is used,

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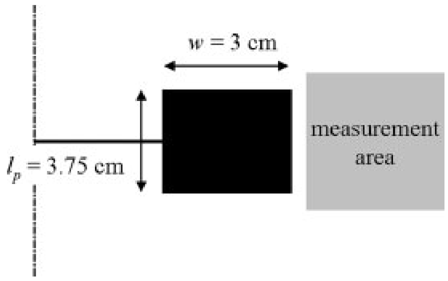

Figure 1. Impeller geometry and measurement area.

measurements need to be carried out in three different kinds of oriented planes (vertical–radial, vertical–tangential, and radial–tangential planes). If the investigation is based on a stereoscopic PIV, two different oriented planes are still necessary. The experimental procedure related to the technique proposed by Jeong and Hussain[15] is thus complex and time-consuming. The goal of this R&D note is to simplify this approach to extend its utilization. Such a simplified procedure was recently used by Ducci and Yianneskis[16] to track macroinstabilities.

A Simplified Procedure

The overall characteristics of the trajectory of the trailing vortices in an impeller stream jet are now globally understood. EBL[5] demonstrated that the displacement of the upper and lower axes follows the trajectory of a fluid particle calculated from the phase-averaged velocity field $\langle U_i^k \rangle$. The trailing vortices move away from the impeller with a main radial–tangential displacement, and they also have a slight vertical motion directed toward the top of the tank because the impeller is located at one third above the bottom of the vessel. An ideal procedure to reduce the number of acquisition planes would be to measure the velocity field in planes perpendicular to the vortex axes, but this is practically inconceivable.

A simpler way is to limit the analysis to the vertical measurement planes, where the trailing vortices can be followed between two successive blades. Only four components of the gradient velocity tensor $\nabla \otimes \langle U \rangle$ are measured: two diagonal components, $\partial\langle U_1\rangle/\partial X_1$ and $\partial\langle U_3\rangle/\partial X_3$, and two nondiagonal components, $\partial\langle U_3\rangle/\partial X_1$ and $\partial\langle U_1\rangle/\partial X_3$, where 1 and 3 denote the radial and axial directions, respectively. Starting from the conservation of mass (equation of continuity) for an incompressible fluid in Cartesian coordinates, the third diagonal component can be estimated as follows:

$$\frac{\partial\langle U_2\rangle}{\partial X_2} = -\left(\frac{\partial\langle U_1\rangle}{\partial X_1} + \frac{\partial\langle U_3\rangle}{\partial X_3}\right) \tag{4}$$

The gradient velocity tensor $\nabla \otimes \langle U \rangle$ is thus expressed as

$$\nabla \otimes \langle U \rangle = \begin{bmatrix} \frac{\partial\langle U_1\rangle}{\partial X_1} & 0 & \frac{\partial\langle U_1\rangle}{\partial X_3} \\ 0 & -\left(\frac{\partial\langle U_1\rangle}{\partial X_1} + \frac{\partial\langle U_3\rangle}{\partial X_3}\right) & 0 \\ \frac{\partial\langle U_3\rangle}{\partial X_1} & 0 & \frac{\partial\langle U_3\rangle}{\partial X_3} \end{bmatrix} = \begin{bmatrix} a & 0 & d \\ 0 & -(a+c) & 0 \\ e & 0 & c \end{bmatrix} \tag{5}$$

The $S^2 + \Omega^2$ tensor is deduced

$$S^2 + \Omega^2 = \begin{bmatrix} a^2 + de & 0 & (1/2)(a+c)(d+e) \\ 0 & (a+c)^2 & 0 \\ (1/2)(a+c)(d+e) & 0 & c^2 + de \end{bmatrix} \tag{6}$$

The eigenvalues of $S^2 + \Omega^2$ are calculated from

$$\det|S^2 + \Omega^2 - \lambda I| = 0 \tag{7}$$

The three eigenvalues are

$$\begin{aligned} \lambda_1' &= (a+c)^2 \\ \lambda_2' &= \frac{a^2 + 2de + c^2 + \sqrt{\Delta}}{2} \\ \lambda_3' &= \frac{a^2 + 2de + c^2 - \sqrt{\Delta}}{2} \end{aligned} \tag{8}$$

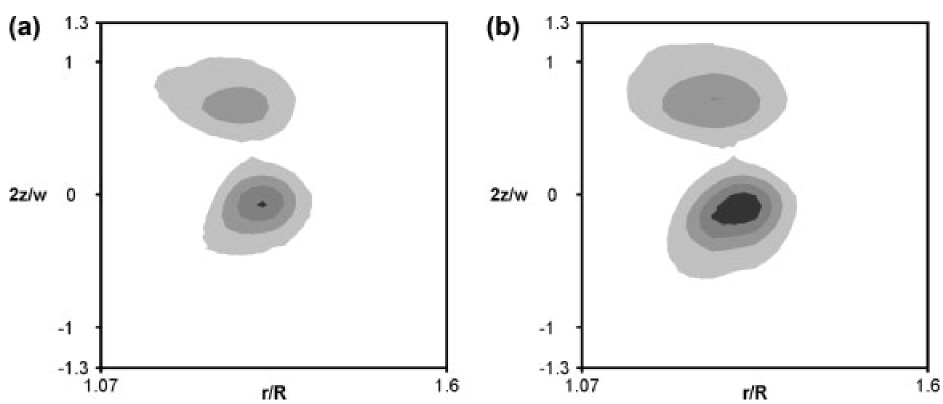

Figure 2. Location of the trailing vortices in vertical plane located at an angular position of 30° behind an impeller blade.

(a) λ_2 estimated from the 2D procedure; (b) λ_2 estimated from the 3D procedure. λ_2(s^{-2}): □: 0 – 7500; ■: −7500 – 0; ■: −15000 – −7500; ■: −225000 – −15000; ■: −30000 – −22500.

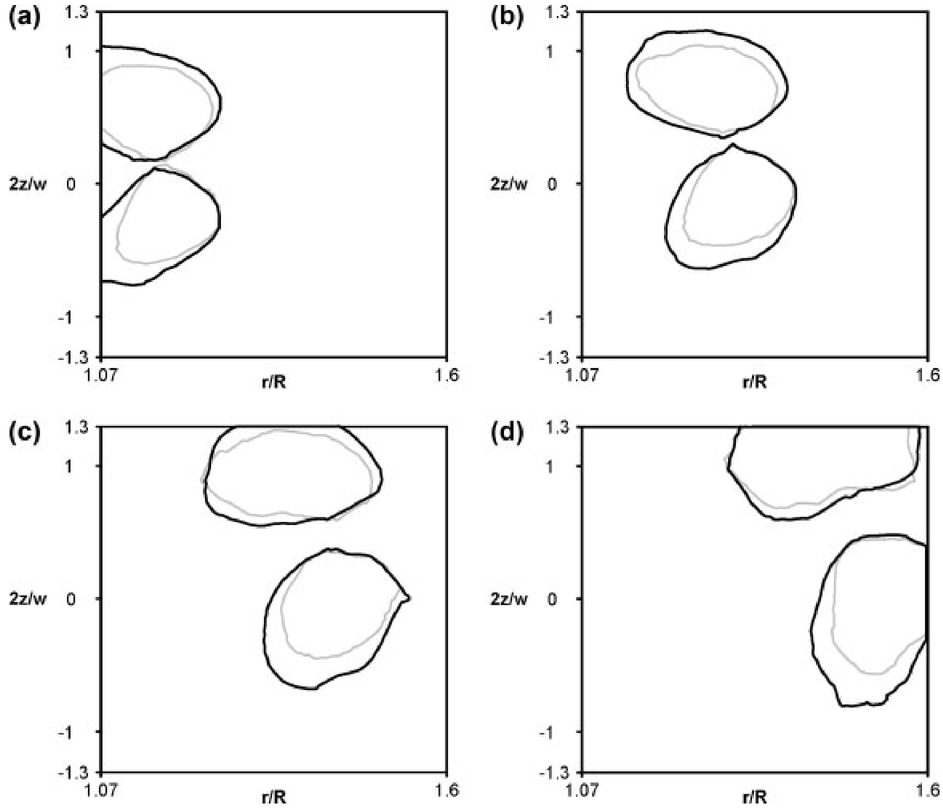

Figure 3. Contour of the trailing vortices derived from 3-D (—) and 2-D (—) procedures.

Angular position: (a) 20°; (b) 30°; (c) 40°; (d) 50°.

with

$$\Delta = (a - c)^2 + (a + c + d + e)^2 \tag{9}$$

Because λ'_1 is positive and $\lambda'_2 > \lambda'_3$, a vortex core corresponds to $\lambda_2 = \lambda'_2 < 0$.

This simplified 2-D procedure based on vertical measurement planes will be tested against the entire 3D procedure based on nine different components of the gradient velocity tensor. The analysis will be performed from data used in our previous article (EBL[5]). 2-D PIV experiments were carried out in a tank stirred by a Rushton turbine. A standard configuration was used [impeller diameter $d = (1/3)T$, where T is the tank diameter]. An extensive description of experimental details (tank, PIV technique, measurement plane location, data acquisition, and analysis) is given by Escudié and Liné.[4] The measurement area is limited to a region close to the Rusthon turbine tip, for a normalized radial position r/R ranging between 1.06 and 1.6 and a normalized vertical position $2z/w$ ranging between -1.3 and 1.3, where R and w are the turbine radius and the blade height, respectively (Figure 1). The data were phase-averaged over 60° (angle between two adjacent blades) with a resolution of 1°.

Trailing Vortex Identification

According to Jeong and Hussain,[15] a vortical structure is characterized by negative λ_2 eigenvalues. Figures 2a and 2b present, for the simplified 2-D and the entire 3-D procedures, respectively, the distribution of the eigenvalue λ_2 in a vertical plane located at an angle of 30° behind a blade. The simplified procedure is able to catch both the trailing vortices generated by the Rushton turbine. Compared to the 3-D procedure (that is, the reference procedure), the size of each vortical structure evaluated from the 2-D procedure is slightly smaller. In addition, the absolute $|\lambda_2|$ within the vortex is underestimated with the 2-D procedure. Given that Escudié[17] demonstrated that the $|\lambda_2|$ value at the center of the vortex core is proportional to the velocity circulation within a vortex (Γ), the simplified procedure cannot record the real magnitude of Γ because the trailing vortex is a three-dimensional structure.

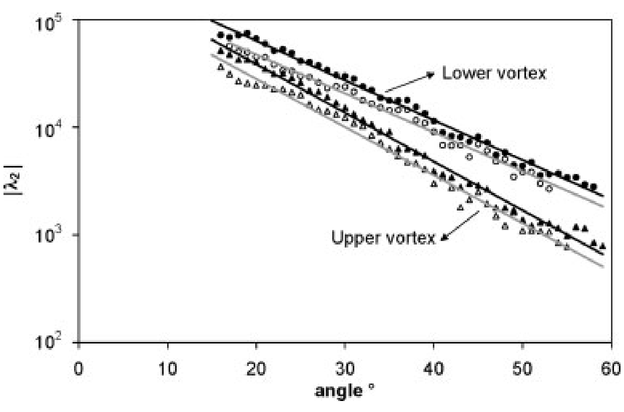

Figure 4. $|\lambda_2|$ estimated at the vortex center from 3-D (—) and 2-D (—) procedures.

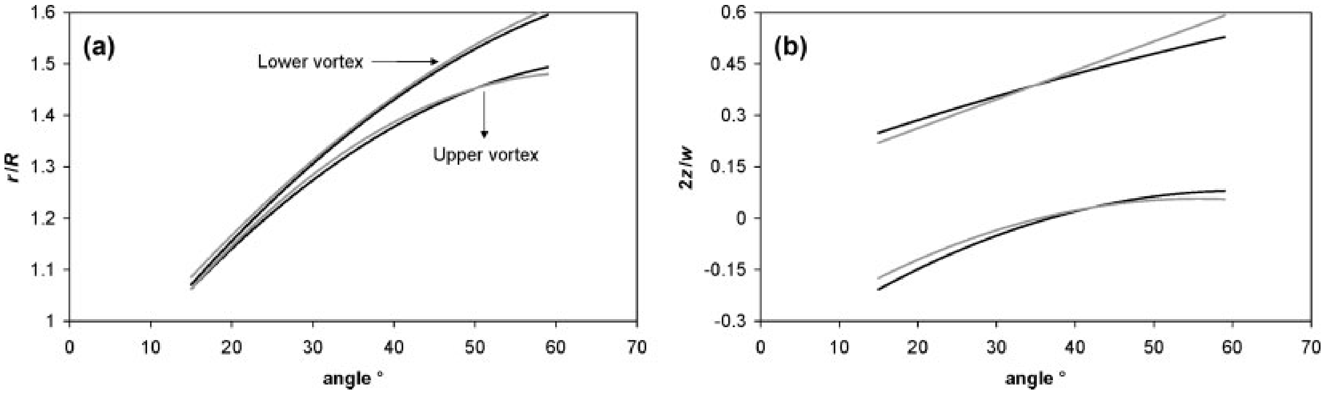

Figure 5. Displacement of the trailing vortices derived from 3-D (—) and 2-D (—) procedures.

(a) Radial displacement; (b) vertical displacement.

Because the vortex region corresponds to $\lambda_2 < 0$, the vortex can be visualized by the surface $\lambda_2 = 0$, corresponding to the contour of the vortex region. Figures 3a–3d present the tip vortex contour, estimated from both procedures for angular positions of the measurement of 20, 30, 40, and 50° behind a turbine blade. The vortex sizes calculated from the 2-D procedure are visually similar to the vortex sizes issued from 3-D data, but slightly smaller.

Figure 4 plots $|\lambda_2|$ estimated at the vortex center (from the 2-D or 3-D procedure) vs. the angular position, for the upper and the lower trailing vortices. The values defining the vortex center correspond to the minimum eigenvalue. $|\lambda_2|$ values, given by both procedures, are relatively close, although the 2-D procedure slightly underestimates $|\lambda_2|$ values by about 20%. The minimum eigenvalue $|\lambda_2|$ decreases with the angular position, similarly to the velocity circulation (EBL[5]).

Figures 5a and 5b plot the radial positions of the center of the vortex and its vertical position, respectively, against the angular position between two successive blades. Once again, the 2-D and 3-D procedures give the same trends in terms of trajectories of trailing vortex centers, the differences being within the mesh resolution of the PIV measurements.

Conclusion

Major parameters for characterizing a trailing vortex are its strength, size, and trajectory. In a previous article, these characteristics were determined from an extensive data acquisition and treatment, based on the Jeong and Hussain[15] method applied to 3-D phase-averaged velocity data in a series of vertical planes between two impeller blades. In the present article, a simplified procedure based on 2-D phase-averaged velocity data has been tested. Results on strength, size, and trajectory of the trailing vortex validate this simplified procedure for identifying trailing vortices in a baffled agitated tank equipped with a Rushton turbine.

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